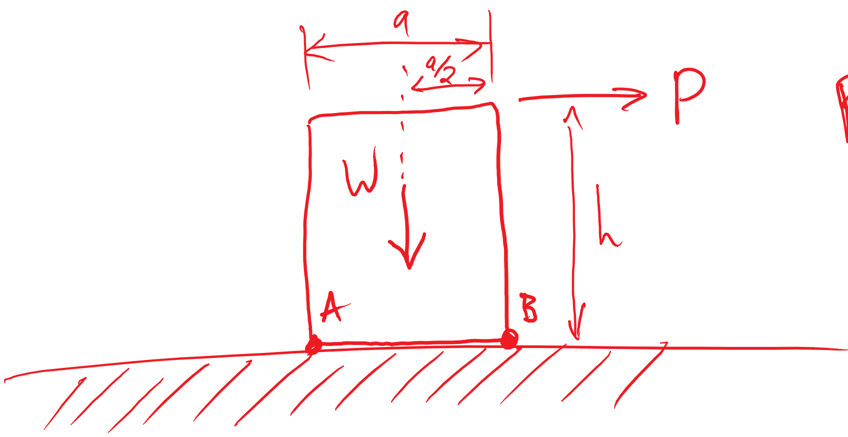
Sunday, April 2, 2017 1:32 PM

Friday April 7th is the last day to switch registration between TAM 210 & 211



Block has weight W
coeff. of static friction is μ_s

Which of the following behaviors is not possible?
(for purposes of TAM 210/211)

A) Slipping (sliding along the surface)
B) Tipping over
C) No motion
D) Melting

For tipping, about which point should we sum moments?

A) A

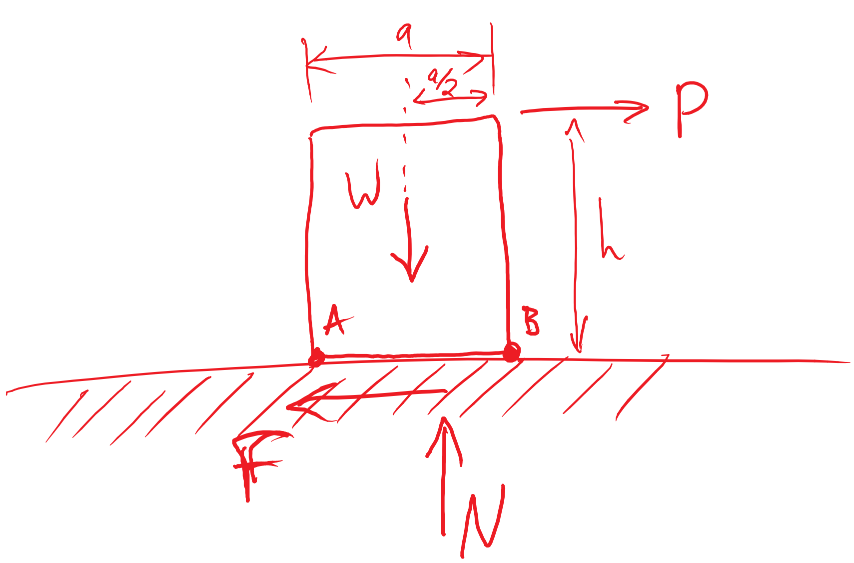
B) B

Impending motion is rotation about B.

Rotation would be clockwise $\Rightarrow (\Sigma M)_B < 0$

$$\Sigma M_B = -P \cdot h + \frac{a}{2} \cdot W < 0$$

$$P > \frac{W \cdot a}{2h}$$ } gives the critical force to tip the object



What about sliding?

$$\Sigma F_x \neq 0$$

$$P - F > 0$$ for motion to the right

$$P > F = \mu \cdot N$$

$$\Sigma F_y = 0 \Rightarrow N = W$$

For sliding: $P > \mu W$ and $P \leq \frac{W \cdot a}{2h}$

What if $P \leq \mu \cdot W$ and $P \leq \frac{W \cdot a}{2h}$?

No motion.

Summary for this block/surface/force

If $P \leq \mu \cdot W$ and $P \leq \frac{W \cdot a}{2h} \Rightarrow$ No motion

If $P > \mu \cdot W$ and $P \leq \frac{W a}{2h} \Rightarrow$ Sliding

If $P > \mu \cdot W$ and $P > \frac{W a}{2h}$ $\Rightarrow$ Tipping and sliding, actual motion requires analysis beyond scope of TAM 210/211

If $P \leq \mu W$ and $P > \frac{W \cdot a}{2h}$ $\Rightarrow$ Tipping about point B

[Student_Chapter9_Centroid.pptx]

Monday, April 3, 2017 1:00 AM

Chapter 9: Center of Gravity and Centroid

Center of Gravity location is very important.

C.G. gives the location where the distributed action of gravity acts.

Center of gravity

To design the structure for supporting a water tank, we will need to know the weight of the tank and water as well as the locations where the resultant forces representing these distributed loads act.

How can we determine these resultant weights and their lines of action?

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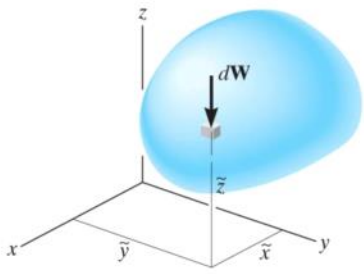
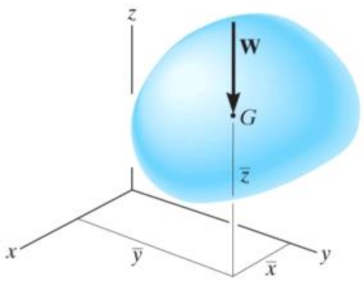
Center of gravity



A body is composed of an infinite number of particles, and so if the body is located within a gravitational field, then each of these particles will have a weight dW.

The **center of gravity (CG)** is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.



From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

If dW is located at point $(\tilde{x}, \tilde{y}, \tilde{z})$ then

position variables (note: tilde "~")

$$\bar{x}\,W = \int \tilde{x}\,\mathrm{dW}$$
$$\bar{y}\,W = \int \tilde{y}\,\mathrm{dW}$$
$$\bar{z}\,W = \int \tilde{z}\,\mathrm{dW}$$

$(\bar{x}, \bar{y}, \bar{z})$ is the location of the C.G.

$\bar{x}, \bar{y}, \bar{z}$ are not variable

$$\bar{x} = \frac{\int \tilde{x} \cdot dW}{W} = \frac{\int \tilde{x} \cdot dW}{\int dW}$$

if W has dimensions of force, what are dim's of

$\int dW$? $\Rightarrow$ force

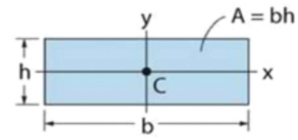
$\int x \cdot dW$? $\Rightarrow$ force $\times$ length

$\frac{\int x\,dW}{\int dW}$? $\Rightarrow$ length

Center of Mass	Center of Volume	Center of Area
m is mass	3D bodies without axes of symmetry	2D, if the body has an axis of symmetry
$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm}$	$\bar{x} = \frac{\int \tilde{x}\, dV}{\int dV}$	$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA}$
$\bar{y} = \frac{\int \tilde{y}\, dm}{\int dm}$	$\bar{y} = \frac{\int \tilde{y}\, dV}{\int dV}$	$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$
$\bar{z} = \frac{\int \tilde{z}\, dm}{\int dm}$	$\bar{z} = \frac{\int \tilde{z}\, dV}{\int dV}$	$\bar{z} = \frac{\int \tilde{z}\, dA}{\int dA}$

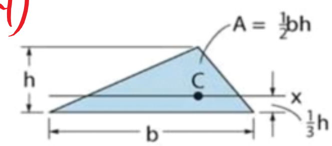
1D: $\bar{x} = \frac{\int_0^L x \cdot dx}{\int_0^L dx}$

Centroid



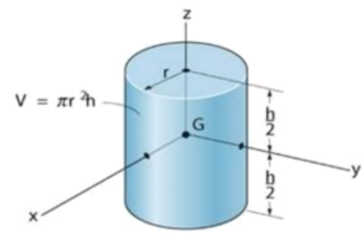
Rectangular area

The centroid, C, is a point defining the geometric center of an object. (useful if uniform density)



Triangular area

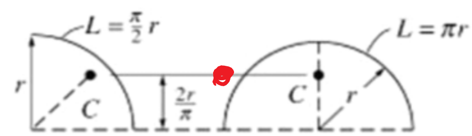
The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogenous (density or specific weight is constant throughout the body).



Cylinder

If an object has an axis of symmetry, then the centroid of object lies on that axis.

In some cases, the centroid may not be located on the object.



Quarter and semicircle arcs

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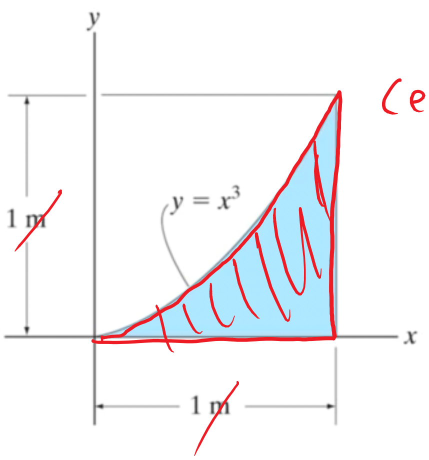
Find the centroid of the area below



centroid: $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{\int x \cdot dA}{\int dA} \qquad \bar{y} = \frac{\int y \cdot dA}{\int dA}$$

$$dA = y \cdot dx$$

$$A = \int dA = \int y \cdot dx = \int_0^1 x^3 \cdot dx = \frac{1}{4} x^4 \Big|_0^1 = \frac{1}{4}$$

$$\bar{x} = \frac{\int x\, dA}{\int dA} = \frac{\int x\, dA}{1/4} = 4 \cdot \int x\, dA$$

$$= 4\int_0^1 x \cdot y \cdot dx = 4\int_0^1 x \cdot x^3 \cdot dx$$

$$= 4\int_0^1 x^4\, dx = 4\left(\frac{x^5}{5}\right)\Big|_0^1 = \frac{4}{5}$$

$$\bar{x} = 4/5$$

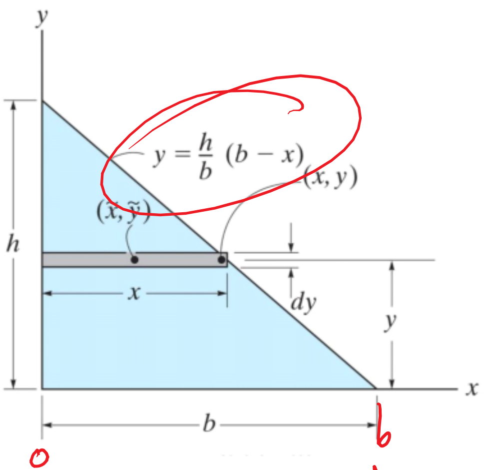
$$\bar{y} = \frac{\int y \cdot dA}{\int dA} = \frac{\int y\, dA}{(1/4)} = 4\int_0^1 y \cdot y\, dx$$

$$= 4\int_0^1 x^3 \cdot x^3 \cdot dx = 4\int_0^1 x^6 \cdot dx = 4 \cdot \frac{x^7}{7}\Big|_0^1 = \frac{4}{7}$$

$\bar{y} = 4/7$

$$\boxed{(\bar{x}, \bar{y}) = \left(\frac{4}{5}, \frac{4}{7}\right) m}$$

Find the centroid of the area below



find $\bar{x}$

$$y = \frac{h}{b}(b - x)$$

$$y = h\left(1 - \frac{x}{b}\right)$$

$$\bar{x} = \frac{\int x \cdot dA}{\int dA}$$

$$\int dA = \int_0^b y \cdot dx = \int_0^b h \cdot \left(1 - \frac{x}{b}\right) dx$$

$$= h \int_0^b \left(1 - \frac{x}{b}\right) dx = h\left(x - \frac{x^2}{2b}\right)\Big|_0^b$$

$$= h \cdot \left(b - \frac{b^{\cancel{2}}}{2\cancel{b}}\right) = h \cdot \frac{b}{2}$$

$$= \frac{1}{2} \cdot b \cdot h$$ Area of a triangle

$$\bar{x} = \frac{\int x\, dA}{\int dA} = \frac{\int_0^b x \cdot h \cdot \left(1 - \frac{x}{b}\right) dx}{\frac{1}{2} b h} = \frac{2\cancel{h}}{b\cancel{h}} \int_0^b \left(x - \frac{x^2}{b}\right) dx$$

$$= \frac{2}{b}\left(\frac{x^2}{2} - \frac{x^3}{3b}\right)\Big|_0^b$$

$$= \frac{2}{b}\left(\frac{b^2}{2} - \frac{b^{\cancel{3}\,2}}{3\cancel{b}}\right) = \frac{2}{\cancel{b}}\left(\frac{3b^{\cancel{2}}}{6} - \frac{2b^{\cancel{2}}}{6}\right)$$

$$= \frac{2b}{6} = \frac{b}{3}$$

$$\boxed{\bar{x} = \frac{b}{3}}$$

Similarly, it can be shown that $\boxed{\bar{y} = \frac{h}{3}}$